PMM U.S.S.R.,Vol.54,No.4,pp.501-507,1990
Printed in Great Britain

0021-8928/90 $10.00+0.00

ANALYSIS OF BOX-LIKE SHELLS OF RECTANGULAR CROSS-SECTION*

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A method of analysing box-like shells, based on reducing the problem to the problem of the combined planar and flexural states of a plate with a defect, for which methods of solution are given in /1/, is proposed (the defect is understood to be a line for which jumps in the force of displacement occur when it is crossed). It is shown that for small thicknesses the solution of the problem of the state of stress of a box-like shell reduces to the sequential solution of two problems (flextural and planar), to within terms of a higher order of smallness. The results of calculating the bending moments and stress in the shell are represented in the form of graphs and tables.

Box-like shells are analysed in /2-5/ using the method of homogeneous solutions, that is effective for particular loading cases or for determining the natural vibrations frequencies. The method used here /6-7/ enables exact solutions to be obtained for an arbitrary load and significantly simplifies the formulation of the problem and the appropriate computations.

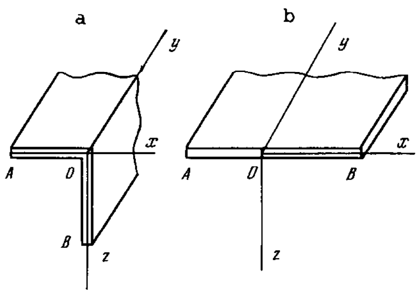

Fig.1

1. Let us examine the state of stress of a shell consisting of two strip-like infinitely long plates of width $OA = a_*, OB = b_*$ joined at right angles and subjected to an arbitrary load (Fig.1a). For simplicity, we will assume that the plates are of identical thickness and are made of the same material. The problem reduces to seeking the solution of the following system of differential equations:

$$D\Delta^2 w_1(x, y) = Z_1(x, y) \tag{1.1}$$

$$\frac{\partial\sigma_x^{(1)}}{\partial x} + \frac{\partial\tau_{xy}^{(1)}}{\partial y} + X_1(x, y) = 0, \quad \frac{\partial\tau_{xy}^{(1)}}{\partial x} + \frac{\partial\sigma_y^{(1)}}{\partial y} + Y_1(x, y) = 0$$

$$\Delta(\sigma_x^{(1)} + \sigma_y^{(1)}) = -(1+\nu)\left(\frac{\partial X_1}{\partial x} + \frac{\partial Y_1}{\partial y}\right); \quad -a < x < 0, \quad -\infty < y < \infty$$

$$D\Delta^2 u_2(y, z) = X_2(y, z)$$

$$\frac{d\sigma_y^{(2)}}{\partial y} + \frac{\partial\tau_{yz}^{(2)}}{\partial z} + Y_2(y, z) = 0, \quad \frac{\partial\tau_{yz}^{(2)}}{\partial y} + \frac{\partial\sigma_z^{(2)}}{\partial z} + Z_2(y, z) = 0$$

$$\Delta(\sigma_z^{(2)} + \sigma_y^{(2)}) = -(1+\nu)\left(\frac{\partial Y_2}{\partial y} + \frac{\partial Z_2}{\partial z}\right); \quad 0 < z < b, \quad -\infty < y < \infty \tag{1.2}$$

that satisfies the boundary conditions $U_k^{(j)} = 0$ on the forces $x = -a$ $(j = 1)$ and $z = b$ $(j = 2)$

$$U_0^{(1)}[w_1] \equiv w_1 - k_0^{(1)} V_x^{(1)} = 0, \quad U_0^{(2)}[u_2] \equiv u_2 + k_0^{(2)} V_z^{(2)} = 0 \tag{1.3}$$

*Prikl.Matem.Mekhan.,54,4,605-612,1990

$$U_2^{(1)}[w_1] \equiv M_x^{(1)} - k_2^{(1)}\varphi_x^{(1)} = 0, \quad U_2^{(2)}[u_2] \equiv M_z^{(2)} + k_2^{(2)}\varphi_z^{(2)} = 0$$
$$U_1^{(1)}[u_1, v_1] \equiv \sigma_x^{(1)} - k_1^{(1)}u_1 = 0, \quad U_1^{(2)}[w_2, v_2] = \sigma_z^{(2)} + k_1^{(2)}w_2 = 0 \tag{1.4}$$
$$U_3^{(1)}[u_1, v_1] \equiv v_1 - k_3^{(1)}\tau_{xy}^{(1)} = 0, \quad U_3^{(2)}[w_2, v_2] \equiv v_2 + k_3^{(2)}\tau_{zy}^{(2)} = 0$$

and the joining conditions at $x = z = 0$, $-\infty < y < \infty$ ensuring equality of the forces and displacements at the shell edge

$$w_1 = D_1\varepsilon^2 w_2, \ u_2 = D_1\varepsilon^2 u_1, \ v_1 = v_2, \ \varphi_x^{(1)} = -\varphi_z^{(2)} \tag{1.5}$$
$$V_x^{(1)} = \sigma_z^{(2)}, \quad M_x^{(1)} = -M_z^{(2)}, \quad \tau_{xy}^{(1)} = \tau_{zy}^{(2)}, \quad V_z^{(2)} = \sigma_x^{(1)}$$

We here introduce dimensionless quantities that are denoted by the same letters as their corresponding physical quantities but will be marked with an asterisk unlike the latter

$$(w_1, \ u_2) = D_1\varepsilon^3 (w_{1*}, \ u_{2*})/a_*, \ (v_j, \ u_1, \ w_2) = \varepsilon\,(v_{j*}, \ u_{1*}, \ w_{2*}) \,/\, a_* \tag{1.6}$$
$$(x, \ y, \ z) = (x_*, \ y_*, \ z_*)/a_*, \ M_n^{(j)} = M_{n*}^{(j)}/(E_* a_*^2), \ V_n^{(j)} = V_{n*}^{(j)}/(E_* a_*)$$
$$\varphi_n^{(j)} = \varphi_{n*}^{(j)} D_1\varepsilon^3, \ (X_1, \ Z_2, \ Y_j) = h_* \,(X_{1*}, \ Z_{2*}, \ Y_{j*}) \,/\, E_*$$
$$(Z_1, \ X_2) = (Z_{1*}, \ X_{2*}) \,/\, E_*, \ \sigma_n^{(j)} = \varepsilon\sigma_{n*}^{(j)}/E_*, \ \tau_{ns}^{(j)} = \varepsilon\tau_{ns*}^{(j)} \,/\, E_*$$
$$D_1 = D_*/(E_* h_*^3), \ b = b_*/a_*, \ \varepsilon = h_*/a_*, \ a = E = D = 1, \quad j = 1, 2$$

The subscripts *1* and *2* correspond to quantities on the horizontal and vertical plates: u_j, v_j, w_j are the displacements of points of the plates in the directions of the x, y, z axes, $M_n^{(j)}, V_n^{(j)}, \varphi_n^{(j)}$ are the bending moment, generalized transverse force, and slope of the plate, $\sigma_n^{(j)}, \tau_{ns}^{(j)}$ are the normal and tangential stresses, X_j, Y_j, Z_j are loadings acting in the directions of the corresponding axes, and h, ν, E, D are the thickness, Poisson's ratio, Young's modulus, and the cylindrical stiffness of the plates.

The operators of the boundary conditions (1.3) and (1.4) describe the conditions of elastic support of the contour with compliance coefficients $k_n^{(j)}$. We note that the case $k_n^{(j)} = \infty$ (here and henceforth, $n = 0, 1, 2, 3$) corresponds to symmetric loading of the box relative to the j-th face while $k_n^{(j)} = 0$ is skew-symmetric. Therefore, the problem of the state of stress of a box-like structure reduces to integrating a system of differential equations with total order 16 and satisfying the corresponding number of boundary conditions.

Significant difficulties are inevitable for the direct solution of the problem because of the awkwardness of the computations and the calculation procedures. Certain simplifications can be achieved in special cases. For instance, if $a = b, k_n^{(1)} = k_n^{(2)}$, the problem can be separated into a sum of the problems of the symmetric and skew-symmetric loading of a corner structure (Fig.1a), each of which reduces to solving a problem concerning the planar-bending state of stress of a strip that is half of the corner struture. It is here necessary to solve half the differential equations, system (1.1), say, that should satisfy the boundary conditions (1.3) and (1.4) for $x = -a$ and the joining conditions (1.5), where the latter will have the following form:

in the symmetric case

$$x = 0, \quad w_1 = -D_1\varepsilon^2 u_1, \quad V_x^{(1)} = \sigma_x^{(1)}, \quad \varphi_x^{(1)} = \tau_{xy}^{(1)} = 0$$

in the skew-symmetric case

$$x = 0, \quad w_1 = D_1\varepsilon^2 u_1, \quad V_x^{(1)} = -\sigma_x^{(1)}, \quad M_x^{(1)} = v_1 = 0$$

But, as will be shown below, even in these cases the approach proposed in this paper will enable the solution to be simplified considerably.

2. Let us alter the formulation of the problem by introducing the new functions

$$(u, v, w)(x, y) = \begin{cases} (u_1, v_1, w_1)(x, y), & x < 0 \\ (w_2, v_2 - u_2)(y, x), & x > 0 \end{cases} \tag{2.1}$$
$$(\sigma_x, \tau_{xy}, \sigma_y)(x, y) = \begin{cases} (\sigma_x^{(1)}, \tau_{xy}^{(1)}, \sigma_y^{(1)})(x, y), & x < 0 \\ (\sigma_z^{(2)}, \tau_{zy}^{(2)}, \sigma_y^{(2)})(y, x), & x > 0 \end{cases}$$
$$(X, Y, Z)(x, y) = \begin{cases} (X_1, Y_1, Z_1)(x, y), & x < 0 \\ (Z_2, Y_2, -X_2)(y, x), & x > 0 \end{cases}$$

Replacement of the unknowns (2.1) corresponds to the following operations: imaginary rotation of the dihedral angle AOB (Fig.1a) and reversal of the sign of the plate deflection YOB while conserving the joining conditions. We consequently obtain the problem of a plate with a defect along the y axis (Fig.1b). Such a problem reduces to integration of the following system of equations

$$D\Delta^2 w(x, y) = Z(x, y) \tag{2.2}$$

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + X = 0, \quad \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + Y = 0$$

$$\Delta(\sigma_x + \sigma_y) = -(1+\nu)\left(\frac{\partial Y}{\partial y} + \frac{\partial X}{\partial x}\right); \quad -0 < x < b, \; x \neq 0, \; -\infty < y < \infty$$

satisfying the homogeneous boundary conditions

$$U_{2k-1}^{(j)}[w] = U_{2k}^{j}[u, v] = 0, \; k, j = 1, 2 \tag{2.3}$$

and the conditions on the defect $x = 0, \; -\infty < y < \infty$

$$\langle v \rangle = \langle \tau_{xy} \rangle = \langle \varphi_x \rangle = \langle M_x \rangle = 0 \tag{2.4}$$

$$\varepsilon^2 D_1 \langle u \rangle = -(w_+ + w_-), \; \langle w \rangle = \varepsilon^2 D_1 (u_+ + u_-)$$

$$\langle \sigma_x \rangle = -[(V_x)_+ + (V_x)_-], \quad \langle V_x \rangle = (\sigma_x)_+ + (\sigma_x)_-$$

$$(F_\pm = F(\pm 0), \; \langle F \rangle = F_- - F_+)$$

The advantage of such an approach is that firstly, the number of differential equations being solved is halved, secondly, the joining conditions are simplified, thirdly, methods of solving both planar and flexural problems for plates with a defect are well developed at the present time and are elucidated in the monograph /1/. We also note that such a formulation should be more convenient than the traditional one /3/ is applying the method of boundary elements.

After application of the Fourier transform in the variable y

$$\begin{Vmatrix} u_\alpha & v_\alpha & w_\alpha & \varphi_\alpha \\ M_{x\alpha} & V_{x\alpha} & \sigma_{x\alpha} & \sigma_{y\alpha} \\ \tau_\alpha & Z_\alpha & X_\alpha & Y_\alpha \end{Vmatrix} = \int_{-\infty}^{\infty} \begin{Vmatrix} u & v & w & \varphi_x \\ M_x & V_x & \sigma_x & \sigma_y \\ \tau_{xy} & Z & X & Y \end{Vmatrix} e^{i\alpha y}\, dy \tag{2.5}$$

problem (2.2)-(2.4) reduces to a one-dimensional discontinuous boundary-value problem

$$L^2 f_\alpha^\pm(x) = q_\alpha^\pm(x), \; -a < x < 0 \wedge 0 < x < b \tag{2.6}$$

with the homogeneous boundary conditions

$$U_{1j}^\pm [f_\alpha^\pm] = U_{2j}^\pm [f_\alpha^\pm] = 0, \; j = 1, 2 \tag{2.7}$$

$$S_2^+ f_\alpha^+ = S_1^+ f_\alpha^+ = S_2^- f_\alpha^- = S_1^- f_\alpha^- = 0, \; S_0^+ f_\alpha^+ = H_3^- f_\alpha^-, \; H_0^+ f_\alpha^+ = -S_3^- f_\alpha^- \tag{2.8}$$

$$[S_3^+ f_\alpha^+]\, \varepsilon^2 D_1 = \alpha^4 [H_0^- f_\alpha^-], \; [H_3^+ f_\alpha^+]\, \varepsilon^2 D_1 = \alpha^4 [S_0^- f_\alpha^-]$$

Here

$$f_\alpha^+(x) = \sigma_{x\alpha}, \; f_\alpha^-(x) = w_\alpha(x), \; q_\alpha^+(x) = -R_3 X_\alpha - i\alpha R_2^+ Y_\alpha, \; q_\alpha^-(x) = Z_\alpha \tag{2.9}$$

and the differential operators

$$R_k^\pm f = \frac{\partial^k f}{\partial x^k}, \quad k = 0, 1, \quad R_2^\pm f = [L + (1 \pm \nu)\alpha^2] f \tag{2.10}$$

$$R_3^\pm f = \frac{\partial}{\partial x}[L - (1 \pm \nu)\alpha^2] f, \quad Lf = \frac{d^2 f}{dx^2} - \alpha^2 f$$

$$Sf = (T^- - T^+) f, \; Hf = (T^- + T^+) f, \; T^\pm f = f(\pm 0)$$

$$S_k^\pm f = S[R_k^\pm f], \; H_k^\pm f = H[R_k^\pm f], \; T_k^\pm f = T[R_k^\pm f]$$

are introduced.

The transforms of the fundamental elastic quantities

$$-\alpha^4 u_\alpha = R_3^+ f_\alpha^+ + \left[\frac{d^2}{dx^2} - 2(1+\nu)\alpha^2\right] X_\alpha + i\alpha \frac{dY_\alpha}{dx}$$

$$-i\alpha^3 v_\alpha = R_3^+ f_\alpha^+ + \frac{dX_\alpha}{dx} + i\alpha Y_\alpha, \quad w_\alpha = R_0^- f_\alpha^-, \quad \varphi_\alpha = R_1^- f_\alpha^-$$

$$M_{x\alpha} = -R_2^- f_\alpha^-, \quad V_{x\alpha} = -R_3^- f_\alpha^-, \quad \sigma_{x\alpha} = R_0^+ f_\alpha^+, \quad i\alpha\tau_\alpha = R_1^+ f_\alpha^+ + X_\alpha$$

can be expressed by using the differential operators (2.10) and functions of the boundary conditions

$$U_{1j}^\pm[f_\alpha^\pm] = R_0^\pm f_\alpha^\pm + (-1)^{j+1}\mu_{1j}^\pm R_3^\pm f_\alpha^\pm, \quad U_{2j}^\pm[f_\alpha^\pm] = R_2^\pm f_\alpha^\pm + (-1)^{j+1}\mu_{2j}^\pm R_1^\pm f_\alpha^\pm$$

$$(\mu_{1j}^- = k_0^{(j)}, \quad \mu_{1j}^+ = \alpha^{-4} k_1^{(j)}, \quad \mu_{2j}^- = k_2^{(j)}, \quad \mu_{2j}^+ = \alpha^2 k_3^{(j)})$$

can be transformed, where $j = 1$ corresponds to the face $x = -a$ and $j = 2$ to the face $x = b$.

We will seek the solution of the discontinuous boundary-value problem (2.6)-(2.8) according to the scheme elucidated in /7/, in the form

$$f_\alpha^\pm(x) = f_q^\pm(x) + \sum_{i=0}^{3} (-1)^i f_i^\pm T_{3-i}^\pm [G_\alpha^\pm(x,t)] \quad (f_i^\pm = S_i^\pm f_\alpha^\pm) \tag{2.11}$$

where $f_i^\pm$ $(i = 0, 1, 2, 3)$ are unknown jumps of the function $f_\alpha^\pm(x)$, $G_\alpha^\pm(x, t)$ are Green's functions of the boundary-value problem (2.6), (2.7), while $f_q^\pm$ is a particular solution given by the relation

$$f_q^\pm = \int_{-a}^{b} q_\alpha^\pm(t) G_\alpha^\pm(x,t)\, dt$$

Here and everywhere below the operators $T_i^\pm$ are applied in the variable t. It follows from the first four conditions (2.7) that four out of the eight unknown jumps equal zero: $f_1^\pm = f_2^\pm = 0$, while the remaining four are a solution of the system of four linear algebraic equations obtained on substituting (2.11) into the last four conditions (2.8)

$$\begin{Vmatrix} 1 & 0 & -C_{33}^- & C_{30}^- \\ 0 & \varepsilon^2\alpha^{-4}D_1 & -C_{30}^- & C_{00}^- \\ -\varepsilon^2 C_{33}^+ & \varepsilon^2 C_{30}^+ & -\alpha^4 D_1 & 0 \\ -C_{03}^+ & C_{00}^+ & 0 & -1 \end{Vmatrix} \begin{Vmatrix} f_0^+ \\ f_3^+ \\ f_0^- \\ f_3^- \end{Vmatrix} = \begin{Vmatrix} H_{3q}^- \\ H_{0q}^- \\ \varepsilon^2 H_{3q}^+ \\ H_{0q}^+ \end{Vmatrix} \tag{2.12}$$

$$(C_{ij}^\pm = H_i^\pm T_j^\pm [G_\alpha^\pm], \quad H_{iq}^\pm = H_i^\pm f_q^\pm)$$

Therefore, after having solved system (2.12), the solution of the boundary-value problem (2.6)-(2.8) will be

$$f_\alpha^\pm = f_q^\pm(x) + f_0^\pm T_3^\pm [G_\alpha^\pm] - f_3^\pm T_0^\pm [G_\alpha^\pm] \tag{2.13}$$

and this enables the transforms of all the elastic quantities to be obtained, in particular

$$w_\alpha(x) = f_0^- T_3^- [G_\alpha^-] - f_3^- T_0^- [G_\alpha^-] + \int_{-a}^{b} q_\alpha(\xi) G_\alpha^-(x,\xi)\, d\xi$$

$$-i\alpha^3 v_\alpha(x) = f_0^+ R_2^+ T_3^+ G_\alpha^+ - f_3^+ R_2^+ T_0^+ G_\alpha^+ + (1+\nu)\int_{-a}^{b} \{X_\alpha(\xi)[(1+\nu)\alpha^4 G_\alpha^{+\prime}(x,\xi)] + Y_\alpha(\xi)(-i\alpha^3)[2LG_\alpha^+(x,\xi) + (1+\nu)\alpha^2 G_\alpha^+(x,\xi)]\}\, d\xi$$

$$-\alpha^4 u_\alpha(x) = f_0^+ R_3^+ T_3^+ G_\alpha^+ - f_3^+ R_3^+ T_0^+ G_\alpha^+ - \alpha^4(1+\nu)\int_{-a}^{b} \{X_\alpha(\xi)[(1-\nu)G_\alpha^{+\prime}(x,\xi) - 2\alpha^2 g_1'(x,\xi)] +$$

$$Y_\alpha(\xi)[(-i\alpha)G_\alpha^{+\prime}(x,\xi)]\} d\xi$$

$$g_1(x,\xi) = \int_{-a}^{\xi} G_\alpha^+(x,t)\,dt$$

Here the prime and dot denote the derivative with respect to the first and second variables, respectively.

We note that system (2.12) splits into two independent second-order systems in the two pairs of jumps $f_0^\pm$ and $f_3^\pm$ for $a = b$ and $k_n^{(j)} = \infty$ $(n = 0, 1, 2, 3; j = 1, 2)$ in the case of symmetric loading of a square box because $C_{03}^\pm = C_{30}^\pm = 0$. By separating the problem into symmetric and skew-symmetric problems with respect to the x coordinate, it can be achieved that one of these pairs will equal zero identically. In this case the solution of the planar (flexural) problem is expressed in terms of one of the jumps $f_3^+ (f_3^-)$ or $f_0^+ (f_0^-)$ for which the analytical expression is sufficiently simple.

For instance, when a shell is compressed by concentrated forces P_* applied at the centres of the faces $y = 0$, $x = \pm b$, the case of the symmetric problem is

$$f_3^- = P(2\rho)^{-1}(2B + \operatorname{sh} 2B)(\operatorname{sh} B + B \operatorname{ch} B),$$
$$f_3^+ = 2P(\varepsilon\rho)^{-1}\alpha^3 \operatorname{sh}^2 B(\operatorname{sh} B + B\operatorname{ch} B)$$
$$\rho = \varepsilon^2[3(1-\nu^2)]^{-1}\alpha^2 \operatorname{sh}^4 B + (B + \operatorname{sh} B \operatorname{ch} B)^2;$$
$$P = P_*/(E_* a_*^2),\ B = \alpha b$$

3. The scheme mentioned was realized to solve the problem of the symmetric loading of a box shell of rectangular profile by a load of constant intensity q applied perpendicular to the middle surface of the horizontal plates $k_n^{(j)} = \infty$ $(j = 1, 2;\ n = 0, 1, 2, 3)$ for two kinds of loading: a load distribution line $x = -a$, $-l \leqslant y \leqslant l$ parallel to the plate joining line (Problem 1), and a load distribution line $y = 0$, $-a - l \leqslant x \leqslant -a + l$ perpendicular to it (Problem 2). Here $q = q_*/(E_* a_*)$, $l = l_*/a_*$. In this case Green's functions $G_\alpha(x, t)$ of the flexural and planar problems are identical.

Values of M_x and σ_x in different sections were calculated on a computer for different ratios between a, b and l. The representation

$$M_x(x,y) = M_x^a(x,y) + \frac{1}{2\pi}\int_{-\infty}^{\infty} M_{x\alpha}^u(x)\,e^{i\alpha y}\,d\alpha$$

was used here.

Here M_x^a is the explicitly inverted weakly convergent part extracted from the particular solution, whose transform has only a power-law decrease in α, as a result of which its numerical inversion is difficult, $M_{x\alpha}^u(x)$ is a function which decreases exponentially with respect to α whose integral can be efficiently evaluated numerically. Values of the transforms of the jumps $f_k^\pm$ are determined directly here from the solution of system (2.12) while the operators and functionals of Green's functions are conveniently programmed in matrix form.

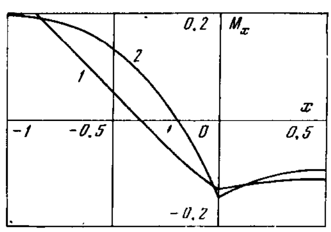

Fig.2

A graph of the dimensionless quantity M_x is shown in Fig.2 for the first kind of loading (curve 1) and the second kind of loading (curve 2) (Problems 1 and 2, respectively) in the section $y = 0$ where M_x has the maximum value for $\nu = 0.4$, $l = 1$, $b = 0.5$, $\varepsilon = 0.01$. Values of the flexural stresses σ_b^1 (at the point $x = -a$, $y = 0$ where they have the maximum values), σ_b^2 (at the point $x = y = 0$ where they are a maximum on the plate joining line) and the maximum planar stresses σ_p (at the point $x = 0$, $y = +0$) are represented in the table for $\nu = 0.4$, $\varepsilon = 0.01$ and a number of values of b, l. The physical quantities are connected by the dimensionless relations

$$\sigma_{x*} = \sigma_x q_* l_*/h_*^2,\ M_{x*} = M_x q_* l_*$$

Agreement between the results of the solution of Problem 1 for large l (the values 5 and 10 wave verified) and the results of solving the plane strain problem (when $l \to \infty$) in a fairly small neighbourhood of y is a confirmation of the validity of the proposed methodology and the numerical computations. Exact agreement within the limits of calculation accuracy is noted in the area $y < 0.7l$. Let us add that the solutions of Problems 1 and 2 become identical as $l \to 0$ (the case of loading by a concentrated force).

b	l	Problem 1			Problem 2		
		σ_b^1	σ_b^2	$-\sigma_p \times 10^5$	σ_b^1	σ_b^2	$-\sigma_p \times 10^5$
0.5	0.1	4.74	1.12	778	4.17	1.07	787
	0.5	2.60	1.01	665	2.05	1.01	885
	1	1.73	0.798	473	1.19	0.940	2330
1	0.1	4.85	0.984	777	4.27	0.973	786
	0.5	2.71	0.875	663	2.13	0.955	883
	1	1.83	0.664	470	1.24	0.880	2330
2	0.1	4.91	0.912	779	4.29	0.939	788
	0.5	2.78	0.801	664	2.14	0.902	886
	1	1.90	0.597	470	1.27	0.743	2330

4. The solution constructed above for the problem of a box-like shell can be simplified assuming the parameter ε to be small. We note that all the elastic quantities in the formulation (1.1)-(1.5) have the same order of smallness in the parameter ε. And if we pass to the limit in (1.5) as $\varepsilon \to 0$, we obtain $w_1 = u_2 = 0$ and problem (1.1)-(1.5) splits into two sequentially solvable problems.

Problem A is to seek the solution of the system of the first two equations (1.1) and (1.2) satisfying the boundary conditions (1.3) and the joining conditions

$$w_1 = u_2 = 0,\ \varphi_x^{(1)} = -\varphi_z^{(2)},\ M_x^{(1)} = -M_z^{(2)} \tag{4.1}$$

Problem B is to seek the solution of the system of the last two equations (1.1) and (1.2) that satisfy the boundary conditions (1.4) and the joining conditions

$$v_1 = v_2,\ \tau_{xy}^{(1)} = \tau_{zy}^{(2)},\ \sigma_x^{(1)} = V_z^{(2)},\ \sigma_z^{(2)} = V_x^{(1)} \tag{4.2}$$

where the values of $V_z^{(2)}$ and $V_x^{(1)}$ are determined from the solution of Problem A.

If the method described in Sect.2 is applied to solve these problems, then Problem A will reduce to the investigation of the bending of a plate having a hinge support

$$\langle \varphi_x \rangle = \langle M_x \rangle = 0,\ w_+ = w_- = 0$$

while Problem B will reduce to an analysis of the plane state of stress of a plate weakened by a defect of the form

$$\langle \tau_{xy} \rangle = \langle v \rangle = 0,\ (\sigma_x)_\pm = \mp (V_x)_\pm$$

The solution of Problems A and B yields an approximate solution of problem (1.1)-(1.5) and its equivalent problem (2.2)-(2.4) apart from a component of the order of smallness $O(\varepsilon^2)$ as $\varepsilon \to 0$. The value of this fact is that standard programs to compute the planar and bending problems of plate theory can be efficiently used to solve problems on the analysis of laminar shells. The solution of Problems A and B is constructed here according to the scheme of Sect.2 by the integral transform method. Then after application of the transformations (2.5) these problems are reduced to the simplified ($\varepsilon = 0$) system (2.12) whose solution has the form

$$f_0^- = 0,\quad f_3^- = (C_{00}^-)^{-1} H_{0q}^-,\quad f_0^+ = H_{3q}^- - C_{30}^+ (C_{00}^-)^{-1} H_{0q}^-$$

$$f_3^+ = (C_{00}^+)^{-1} \{H_{0q}^+ - C_{03}^+ H_{3q}^- + (C_{00}^-)^{-1} [1 - C_{30}^+ C_{03}^-] H_{0q}^-\}$$

Then in the case of a load in the form of a concentrated force with components (P_{x*}, P_{y*}, P_{z*}) applied at the point (ξ, η), the transforms of the bending σ_{1*} and planar stresses σ_{2*} that occur in the plates can be written in the form

$$\varepsilon^2 a_*^2 \sigma_{1*} = 6 P_{z*} e^{i\alpha\eta} R_2^- Q(x, \xi)$$

$$\varepsilon a_*^2 \sigma_{2*} = e^{i\alpha\eta} \{P_{x*} R_3^+ Q(x, \xi) + P_{y*} (-i\alpha) \cdot R_2^+ Q(x, \xi) + P_{z*} (C_{00}^-)^{-2} [C_{00}^+ C_{03}^+ b_{30}(x, \xi) + (C_{30}^+ C_{03}^- - 1) b_{00}(x, \xi) + b_{33}(x, \xi) - C_{03}^+ C_{00}^+ b_{03}(x, \xi)]\}$$

$$Q(x, \xi) = G_\alpha(x, \xi) - G_\alpha(x, 0) G_\alpha(0, \xi) G_\alpha^{-1}(0, 0),$$

$$b_{jk}(x, \xi) = [H_j^- G_\alpha(x, \xi)] [T_k^+ G_\alpha(x, \xi)]$$

where the coefficients $C_{jk}^\pm$ are defined in (2.12) and the operators $R_j^\pm$, $T_j^\pm$, $H_j^\pm$ in (2.10), where the operators are here applied to the second variable. In particular, in the case of Problem 1 the maximum bending stresses are

$$\sigma_* = \frac{6q_*}{\pi a_* \varepsilon^2} \int_0^\infty \frac{\sin \alpha l}{\alpha} \times \frac{(1+\nu) \operatorname{ch} A [B \operatorname{ch} A + \operatorname{ch} B \operatorname{sh}(A+B)] + (1-\nu) A (B + \operatorname{ch} B \operatorname{sh} B + A \operatorname{ch}^2 B)}{B \operatorname{ch}^2 A + A \operatorname{ch}^2 B + \operatorname{ch} A \operatorname{ch} B \operatorname{sh}(A+B)} d\alpha$$

$$(A = \alpha a, \ B = \alpha b)$$

Calculations showed that for $\varepsilon < 0,1$ in the case of Problems 1 and 2 the exact and approximate solutions agree to within three significant figures.

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Translated by M.D.F.